Definition. Let a and d be integers. We say that d divides a (that is, a is a multiple of d), written as $d|a$, if and only if there is an integer k such that $a = kd$.

Example. $5|15$

(1) Determine if the following statements are true.

(a) $2|6$ **(b)** $3|7$ **(c)** $8|4$ **(d)** $8|0$ **(e)** $0|1$ **(f)** $75|50$ **(g)** $10^2|10^3$ **(h)** $-5|20$ **(i)** $8|-24$ **(j)** $-6|-42$ **(k)** $-6|39$ **(l)** $0|0$

(2) If $a|b$ and $b|c$, is it true that $a|c$? Can you give an algebraic argument for this? (Note: this means you cannot use a numeric example to show that this is true, it has to work no matter what values a, b, and c are used.)

(3) If $a|b$ and $a|c$, does it mean that $a|bc$? Why or why not? If it is true, give an algebraic argument. If it is not true, then give a *counterexample*.

(4) If $a|b$ and $a|c$, is it true that $a|b+c$? Why or why not?

(5) If $a|bc$, does it mean that $a|b$ or $a|c$? Why or why not?

(6) In the previous part, can you give some conditions where the statement will *definitely* work?