Turing Machines Examples
Example 1

- Given TM $M_2$ that decides $A = \{ 0^{2^n} | n \geq 0 \}$
  - Language consists of strings of 0’s whose length is a power of 2

- $M_2 =$ “On input string $w$:
  1. Sweep left to right across the tape, crossing off every other 0
  2. If in stage 1 the tape contained a single 0, accept
  3. If in stage 1 the tape contained more than a single 0 and the number of 0s was odd, reject
  4. Return the head to the left-hand end of the tape
  5. Go to stage 1

- At each iteration of stage 1, the number of 0s is cut in half
  - Done by marking every other 0
  - Keeps track of number of 0s on each pass
Example 1

- Formal Definition, $M_2 = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$
  - $Q = \{q_1, \ldots, q_5, q_{accept}, q_{reject}\}$
  - $\Sigma = \{0\}$
  - $\Gamma = \{0, x, _\}$
  - We describe $\delta$ with a state diagram
  - The start, accept, and reject states are $q_1$, $q_{accept}$, and $q_{reject}$, respectively

- Notation: $a \rightarrow b, D$
  - $a = \text{symbol read on tape}$
  - $b = \text{symbol to be written on tape}$
    - May be left blank (does not alter tape)
  - $D = \text{direction the head moves}$
  - Shorthand for $\delta(q,a) = (r,b,D)$
Example 1

Transitions

- Begins by writing a blank on the leftmost 0
  - Marks the start of tape

- State 2, checks if # of 0s is correct
  - If input = 0, marks x, moves right, go to q₃
  - If input = x, moves right
  - If input = ˽, moves right, go to q_{accept}

- State 3, intermediate transition
  - If input = 0, moves right, go to q₄
  - If input = x, moves right
  - If input = ˽, moves left, go to q₅
Example 1

- Transitions
  - State 4, checks if # of 0s is **incorrect**
    - If input = 0, moves right, marks x, go to q₃
    - If input = x, moves right
    - If input = ω, moves right, go to q₃
  - State 5, moves head back to left side
    - If input = 0, moves left
    - If input = x, moves left
    - If input = ω, moves right, go to q₂
Example 1

- Sample input: 0000
- Start configuration: $q_10000$
Example 2

- TM of previous lecture which decides the language $B = \{w#w \mid w \in \{0,1\}^*\}$

- Formal Definition, $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$
  - $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$
  - $\Sigma = \{0,1,\#\}$
  - $\Gamma = \{0,1,\#,x,\$$\}$
  - We describe $\delta$ with a state diagram
  - The start, accept, and reject states are $q_1$, $q_{\text{accept}}$, and $q_{\text{reject}}$, respectively

- Notation: $a,b \rightarrow D$
  - Reads either $a$ or $b$
  - Moves in the D direction

- Reject state is not shown
  - Rejects when reading symbol without a transition
**Example 3**

- $M_3$ decides the language $C = \{a^ib^jc^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$

- $M_3 =$ “On input string $w$:
  1. Scan input from left to right to check if single is a member of $a^+b^+c^+$
     - Reject if not a member
  2. Return head to leftmost end
  3. Cross off an $a$ and scan right until a $b$ occurs
     - Go back and forth between $b$’s and $c$’s while crossing off one of each until all $b$’s are gone
     - If all $c$’s are crossed off and some $b$’s remain, reject
  4. Restore crossed off $b$’s and repeat stage 3 if there is another $a$ to cross off
     - If all $a$’s are crossed off and all $c$’s are crossed off, accept
     - Otherwise, reject”
Example 4

- $M_4$ decides the language

$$E = \{#x_1#x_2# \ldots #x_l | \text{each } x_i \in \{0,1\}^* \text{ and } x_i \neq x_j \text{ for each } i \neq j\}$$

- $M_4 =$ “On input $w$:

1. Place a mark on top of the leftmost tape symbol (ex $\#$).
   - If that symbol was a black, accept
   - If symbol was a $\#$, continue with the next stage
   - Reject, otherwise
2. Scan right to the next $\#$ and place a second mark on top of it
   - If no $\#$ is encountered before a blank symbol, only $x_1$ was present; accept
3. Go back and forth between the two words on the right of the marked hashes
   - If they match reject
4. Move the $2^{nd}$ mark to the next hash symbol
   - If there is no more hash symbols on the right, then move the $1^{st}$ hash symbol to its next one
     - Move the $2^{nd}$ mark to the hash immediately after the $1^{st}$
     - If no hashes are available to move to, all words have been compared; accept
5. Return to stage 3”