Definition. A common divisor of integers a and b is an integer d such that $d|a$ and $d|b$.

Example. 4 is a common divisor of 16 and 32

Definition. The *greatest common divisor* of two integers a and b, not both 0, is the largest integer d such that $d|a$ and $d|b$. The greatest common divisor of two integers a and b is denoted $\gcd(a, b)$.

Example. The greatest common divisor of 32 and 48 is 16.

The **Euclidean Algorithm** can be used to find the gcd of two numbers, when factoring deems difficult. The algorithm runs by repeatedly diving the larger number by the smaller number until there is no remainder.

Example. Find $\gcd(3094, 2513)$. Then

$$\begin{aligned} 3094 &= 1 \cdot 2513 + \mathbf{581} \\ 2513 &= 4 \cdot 581 + \mathbf{189} \\ 581 &= 3 \cdot 189 + \mathbf{14} \\ 189 &= 13 \cdot 14 + \mathbf{7} \\ 14 &= 2 \cdot 7 + \mathbf{0} \end{aligned}$$

In the last statement, since there is no remainder, the remainder before the last statement, 7, is the gcd.

(1) Find the greatest common divisor of each pair of integers by factoring:

(a) 144 and 48
(b) 5280 and 3600
(c) 2^{100} and 100^2
(d) 24^{120} and 120^{24}
(e) $10!$ and 3^{10}
(f) 10^{100} and $100!$

(2) Find the greatest common divisor of each pair of integers using the Euclidean algorithm.

(a) 484 and 451
(b) 5280 and 3600
(c) 3953 and 1829
(d) 144 and 89

(3) Prove that if $a|b$ and $a|c$, and x, y are integers, then $a|bx + cy$.

(4) Prove that if $a|b$ and $b|a$, then $a = b$ or $a = -b$.

(5) Can you give some reasoning why the Euclidean Algorithm works?