1. Find all separable solutions of the partial differential equation

\[ u_x(x, y) + u_y(x, y) = 0 \]

Note that your answer will depend on the separation constant. Which separable solutions satisfy both

\[ \lim_{x \to \infty} u(x, y) = 0 \]

and

\[ \lim_{y \to \infty} u(x, y) = 0 \]

2. Find all separable solutions of the partial differential equation

\[ u_{xx}(x, y) + u_{yy}(x, y) = 0 \]

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3. Let a surface \( S \) be described (by a Monge patch) as the set of points in \( \mathbb{R}^3 \) as

\[ \{(x, y, z) \mid z = f(x, y)\}, \]

for some smooth function \( f \) of two variables. Let \( f_x \) and \( f_y \) be the partial derivatives of \( f \). It can be shown that the Gaussian Curvature, \( K \), for the surface can be computed to be

\[ K = \frac{(f_{xx}f_{yy} - f_{xy}^2)}{(1 + f_x^2 + f_y^2)^2}, \]

and the Mean Curvature, \( H \), for the surface can be computed to be

\[ H = \frac{(1 + f_y^2)f_{xx} + (1 + f_x^2)f_{yy} - 2f_xf_yf_{xy}}{2(1 + f_x^2 + f_y^2)^{3/2}}. \]

Why does any solution to problem 2. have negative Gaussian Curvature? Can the solution ever have a relative maximum or relative minimum? Why or why not?