1. Find the first 4 terms of the Taylor Series for \( f(x) = 1/(1 - x) \) expanded about \( c = 0 \). For which real numbers \( x \) will this Taylor series converge to \( f(x) \)?

2. Give an example of machine arithmetic on three real numbers where \( (a + b) + c \) is not equal to \( a + (b + c) \). Can you find an example of machine arithmetic where \( a + b \) is different from \( b + a \)?

3. Convert the decimal 513.375 to binary.

4. Find what real number is represented by the following single-precision floating point numbers (I have put spaces between the 3 fields and I have not written the understood “1.” before the mantissa):

   a. 0 00000000 000000000000000000000000
   b. 1 10000011 100000000000000000000000
   c. 0 01111111 100100000000000000000000

5. Assume that you are trying to use Newton’s method to find the cube root of \( R > 0 \). What is the recursion relation (which tells what \( x_{n+1} \) is in terms of \( x_n, f(x_n) \), and \( f'(x_n) \)) you should use? What would be a reasonable first guess for \( x_0 \)?

6. Suppose that a function \( f(x) \) has a root at \( r \) and that its derivative \( f'(x) \) has a root very near to \( r \). For example, let \( f(x) = cx^{-1} - c^2 x^{-2} \) where \( c = 0.1 \) (or smaller). In view of our error analysis of Newton’s method, can you safely use it for finding \( r \)? Why or why not?

7. Is it better to compute an expression as \( \frac{\sin^2 x}{x^2} \) or as \( \frac{1 - \cos(2x)}{2x^2} \)? Why?

8. Someone has written the following function. Trace the function call \( \text{cubic}(1.0) \) and find out what will be returned.

   ```c
   double a=1.0, b=-1.0, c=1.0, d=-1.0;
   double cubic(double x) {
       double z;
       z = ((a*x + b)*x + c)*x + d;
       return(z);
   }
   ```

9. If \( f \) is a continuous function on an interval \([a, b]\) and if the sign of \( f(a) \) is different from the sign of \( f(b) \) then there is root \( r \) in the interval. The method of Bisection can be safely used to find an approximate root. Is this the best we can do for a safe method? Why or why not?

10. In polynomial interpolation what advantages (list as many as you can) does Newton’s method have over Lagrange’s method?

11. Use the relationship between divided differences

\[
 f[x_0, x_1, x_2, \ldots, x_k] = \frac{f[x_1, x_2, \ldots, x_k] - f[x_0, x_1, \ldots, x_{k-1}]}{x_k - x_0}
\]

to find the Newton polynomial which interpolates the four points: \( f[-2] = -9 \), \( f[-1] = -15 \), \( f[0] = -5 \), \( f[1] = -3 \), and \( f[2] = 39 \).

12. Why do numerical analysts approximate derivatives as follows

\[
 f'(x) \approx \frac{f(x + h) - f(x - h)}{2h}
\]

and

\[
 f''(x) \approx \frac{f(x + h) - 2f(x) + f(x - h)}{h^2}
\]